High-Rate Analysis of Systematic Lossy Error Protection of a Predictively Encoded Source

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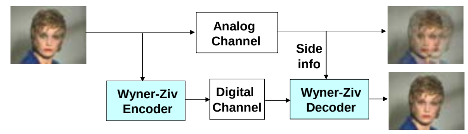
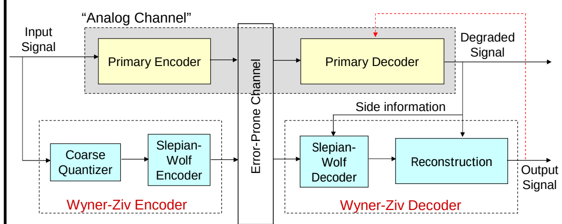
Systematic Lossy Source/Channel Coding



- Errors corrected up to a small residual distortion, hence **lossy** error protection
- Information theoretic optimality conditions
  [Shamai, Verdú, Zamir, 1998], [Steinberg, Merhav, 2004]

Systematic Lossy Error Protection (SLEP)



- Analogous to systematic lossy source/channel coding
- Practical Wyner-Ziv coding by applying Reed-Solomon coding across coarsely quantized redundant video descriptions
  [Rane, Baccichet, Aaron, Girod, 2004-06]

Foreman @ 408 kbps, Wyner-Ziv bit rate = 40 kbps

Symbol error probability = 5 x 10^{-4}

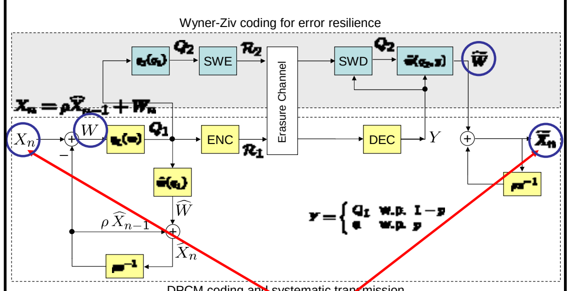
Error-free (QP = 28)
35.7 dB

[illegible]

Outline

- Analysis of SLEP for a first-order Markov source
- Rate-distortion tradeoffs using high-rate quantization theory
- Properties of SLEP
  - Graceful degradation of decoded signal quality
  - Increased robustness compared to lossless correction

SLEP of a First-Order Markov Source



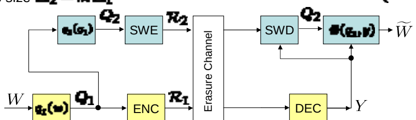
Find the relation between $D = E(X-\hat{X})^2$ and $R = R_1 + R_2$

Wyner-Ziv Coding of Prediction Residual W

$E(W-\hat{\tilde{W}})^2 = \frac{m^2\Delta_1^2}{12}$

step size $\Delta_2 = m\Delta_1$

$\tilde{W} = E(W|Q_2,Y) = \begin{cases} \hat{W} & \text{if } Y = Q_1 \\ \hat{\hat{W}} & \text{if } Y = e \end{cases}$



step size Δ_1 $\quad R_1 = H(Q_1)$

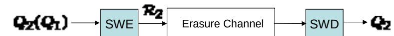
$Y = \begin{cases} Q_1 & \text{w.p. } 1-p \\ e & \text{w.p. } p \end{cases}$

$E(W-\hat{W})^2 = \frac{\Delta_1^2}{12}$

$D_1 \simeq \frac{1}{12} 2^{2h(W)} 2^{-2R_1} = \frac{\Delta_1^2}{12}$

$D_2 = E(W-\tilde{W})^2 \simeq (1-p)D_1 + pm^2 D_1$ where $m \in Z^+$

Bit Rate Required for Erasure Protection



$Y = \begin{cases} Q_1 & \text{w.p. } 1-p \\ e & \text{w.p. } p \end{cases}$

- Minimum bit rate with ideal Slepian-Wolf coding and no erasures

$H(Q_2|Y) = (1-p)\, H(Q_2|Q_1) + p\, H(Q_2|e) = 0 + p\, H(Q_2)$

- To perform Slepian-Wolf coding with erasure protection, need

$R_2 = \frac{1}{1-p} H(Q_2|Y) = \frac{p}{1-p} H(Q_2) \simeq \frac{p}{1-p}(R_1 - \log_2 m)$

$D_2 = E(W-\tilde{W})^2 \simeq \frac{(1-p+pm^2)}{m^2} \frac{1}{12} 2^{2h(W)} 2^{-2R_2\frac{1-p}{p}}$

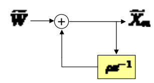
Error Propagation in X

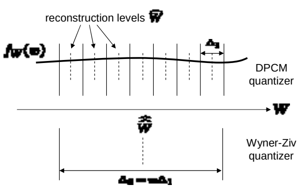
$X_n - \tilde{X}_n = \rho(\hat{X}_{n-1} - \tilde{X}_{n-1}) + (W_n - \tilde{W}_n)$

$D = E(X-\tilde{X})^2$

$= \frac{\rho^2}{1-\rho^2} p\, E(\hat{W} - \hat{\hat{W}})^2 + E(W-\tilde{W})^2$

D_2





Quantization mismatch

$E(\hat{W} - \hat{\hat{W}})^2 = (m^2-1)\frac{\Delta_1^2}{12}$

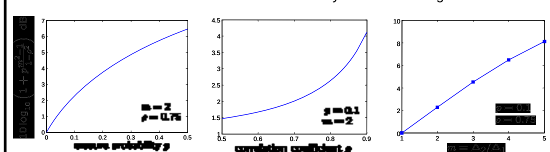
$\simeq (m^2-1)D_1$

Overall MSE Distortion in X

$D = E(X-\tilde{X})^2 \simeq \left(1 + p\frac{m^2-1}{1-\rho^2}\right) m^{-2p} \frac{1}{12} 2^{h(W)} 2^{-2R(1-p)}$

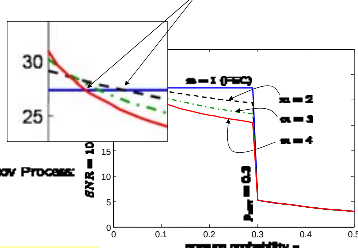
Alternatively, $D = D_1\left(1 + p\frac{m^2-1}{1-\rho^2}\right)$

Additional distortion after Wyner-Ziv decoding



Graceful Degradation

$p = \frac{(1-\rho^2)(m^{2p_{clif}}-1)}{m^2-1} < p_{clif}$ for $m > 1$



First-order Gauss-Markov Process:

$X_n = \rho X_{n-1} + W_n$

$\rho = 0.75, \sigma_W^2 = 5$

Fix $R = R_1 + R_2 = 6$ bits, $p_{clif} = 0.3$
Vary m, Find R_1, R_2, D

Increased Robustness

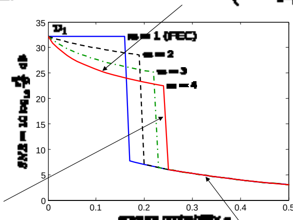
First-order Gauss-Markov Process:

$X_n = \rho X_{n-1} + W_n$

$\rho = 0.75, \sigma_W^2 = 5$

Fix $R_1 = 5$ bits, $R_2 = 1$ bit
Vary m, Find D, p_{clif}

$D = D_1\left(1 + p\frac{m^2-1}{1-\rho^2}\right)$



$p_{clif} = \frac{R_2}{R_1 + R_2 - \log_2 m}$

Cliff occurs because $\sigma_W^2 \gg m^2 D_1$

$D = D_1\left(1 + p\frac{(\sigma_W^2/D_1)-1}{1-\rho^2}\right)$

Conclusions

- SLEP output distortion depends on quality of redundant description, erasure probability, correlation coefficient of source
- Flexible tradeoff between robustness and signal quality by manipulating Wyner-Ziv quantizer
- With identical source/channel bit allocation, SLEP is more robust to channel errors than FEC

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